
UNIVERSITI SAINS MALAYSIA

Final Examination
Academic Session 2007/2008

April 2008

JIM 101 – Calculus
[Kalkulus]

Duration : 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains NINE printed pages before you begin the examination.

Answer ALL questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Jawab SEMUA soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.]

1. (a) Solve the following inequalities

(i) $\frac{1}{3x-1} > \frac{2}{x+5}$

(ii) $|x+3| \geq |x-3|$.

(40 marks)

- (b) Given $f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 5$, form the composition function $g \circ f$ and specify the domain and range of $g \circ f$.

(30 marks)

- (c) Evaluate the following limits if exist

(i) $\lim_{x \rightarrow 1} \frac{1-x^3}{x-1}$

(ii) $\lim_{h \rightarrow 0} \frac{3 - \sqrt{9+h}}{h}$

(iii) $\lim_{x \rightarrow 7^+} \frac{|x-7|}{x-7}$.

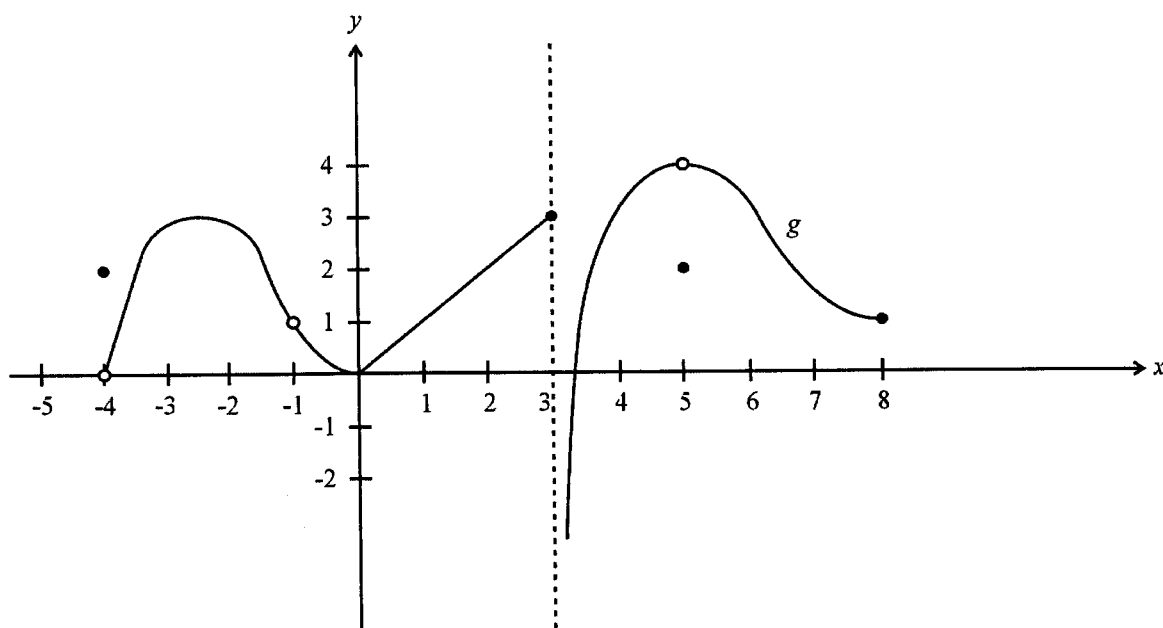
(30 marks)

2. (a) Let $f(x) = \begin{cases} 3-x^2, & x \leq -2 \\ ax+b, & -2 < x < 2 \\ \frac{1}{2}x^2, & 2 \leq x. \end{cases}$

Find a and b such that $\lim_{x \rightarrow -2} f(x)$ and $\lim_{x \rightarrow 2} f(x)$ exist.

(30 marks)

(b)



The graph of g is given in the figure above.

- (i) Determine the intervals on which g is continuous.
- (ii) Determine whether or not the function is continuous at $x = -4$, $x = -1$, $x = 3$, $x = 5$ and $x = 8$. If not, determine the type of discontinuity.

(30 marks)

- (c) Give the necessary and sufficient conditions on a and b for the function

$$f(x) = \begin{cases} ax - b, & x \leq 1 \\ 3x, & 1 < x < 2 \\ bx - a, & 2 \leq x \end{cases}$$

to be continuous at $x = 1$ but discontinuous at $x = 2$.

(40 marks)

3. (a) (i) Use the definition of derivative to find $f'(x)$ if $f(x) = 4x^2 - 3x + 1$.

- (ii) Find a polynomial of degree two such that $f(2) = 2$, $f'(2) = 4$ and $f''(2) = 6$.

(20 marks)

(b) Find $\frac{dy}{dx}$, if

(i) $y = x^{\tan x}$

(ii) $x \cos y + y \cos x = 1$

(iii) $y = \frac{(x^2 + 3)^3 (3x - 4)^4}{\sqrt{x}}$.

(30 marks)

(c) Given $f(x) = x^5 - 5x^3$

(i) Find the interval where f is increasing and the interval where f is decreasing.

(ii) Find the local extremum of f .

(iii) Find the interval where f concave upwards and the interval where f concave downwards.

(iv) Find the reflection points.

(v) Sketch the graph of f

(50 marks)

4. (a) Evaluate

(i) $\int (2x^3 + 1)^3 x^2 dx$

(ii) $\int \frac{x}{3 + x^4} dx$

(iii) $\int_1^e \ln x dx$

(iv) $\int_0^2 \frac{x^2 + 2}{x + 1} dx$

(v) $\int \frac{x dx}{x^2 + 4x - 5}$.

(50 marks)

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- (b) (i) Find the area of the region bounded by the graph of $y = 3^x$, $x + y = 1$ and $x = 1$.
- (ii) Find the equation of a circle that passes through the focus and vertex of the parabola $y = 8x^2$ and is tangent to the parabola at $(0,0)$. (30 marks)
- (c) Prove that the volume of a cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$. (20 marks)

1. (a) Selesaikan ketaksamaan berikut

(i) $\frac{1}{3x-1} > \frac{2}{x+5}$

(ii) $|x+3| \geq |x-3|$.

(40 markah)

(b) Diberi $f(x) = \sqrt{x+1}$ dan $g(x) = x^2 - 5$, bentukkan fungsi gubahan $g \circ f$ dan tentukan domain dan julat $g \circ f$.

(30 markah)

(c) Nilaikan had berikut jika wujud

(i) $\lim_{x \rightarrow 1} \frac{1-x^3}{x-1}$

(ii) $\lim_{h \rightarrow 0} \frac{3 - \sqrt{9+h}}{h}$

(iii) $\lim_{x \rightarrow 7^+} \frac{|x-7|}{x-7}$.

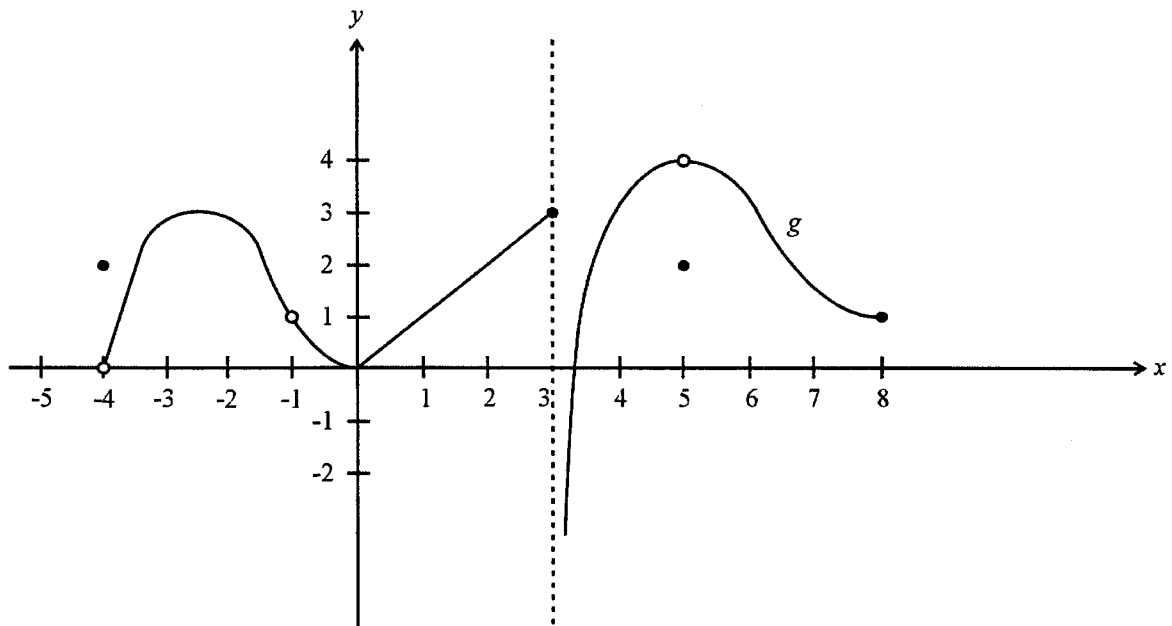
(30 markah)

2. (a) Katakan $f(x) = \begin{cases} 3-x^2, & x \leq -2 \\ ax+b, & -2 < x < 2 \\ \frac{1}{2}x^2, & 2 \leq x. \end{cases}$

Dapatkan nilai a dan b supaya $\lim_{x \rightarrow -2} f(x)$ dan $\lim_{x \rightarrow 2} f(x)$ wujud.

(30 markah)

(b)



Diberi graf fungsi g seperti di atas.

- (i) Tentukan selang di mana g selanjat.
- (ii) Tentukan sama ada fungsi g selanjat pada $x = -4$, $x = -1$, $x = 3$, $x = 5$ dan $x = 8$. Jika tidak, tentukan jenis ketidakselanjatan tersebut.

(30 markah)

(c) Berikan syarat perlu dan cukup terhadap a dan b supaya fungsi

$$f(x) = \begin{cases} ax - b, & x \leq 1 \\ 3x, & 1 < x < 2 \\ bx - a, & 2 \leq x \end{cases}$$

selanjat pada $x = 1$ dan tidak selanjat pada $x = 2$.

(40 markah)

3. (a) (i) Dengan menggunakan takrif terbitan, cari $f'(x)$ jika $f(x) = 4x^2 - 3x + 1$.

(ii) Cari satu polinomial berdarjah dua supaya

$$f(2) = 2, f'(2) = 4 \text{ and } f''(2) = 6.$$

(20 markah)

(b) Cari $\frac{dy}{dx}$, jika

(i) $y = x^{\tan x}$

(ii) $x \cos y + y \cos x = 1$

(iii) $y = \frac{(x^2 + 3)^3 (3x - 4)^4}{\sqrt{x}}$.

(30 markah)

(c) Diberi $f(x) = x^5 - 5x^3$

(i) Cari selang di mana f menaik dan selang di mana f menyusut.

(ii) Cari ekstremum tempatan bagi f .

(iii) Cari selang di mana f cengkung ke atas dan selang di mana f cekung ke bawah.

(iv) Cari titik lengkok balas.

(v) Lakarkan graf f .

(50 markah)

4. (a) Nilaikan

(i) $\int (2x^3 + 1)^3 x^2 dx$

(ii) $\int \frac{x}{3 + x^4} dx$

(iii) $\int_1^e \ln x dx$

(iv) $\int_0^2 \frac{x^2 + 2}{x + 1} dx$

(v) $\int \frac{x dx}{x^2 + 4x - 5}$.

(50 markah)

- (b) (i) Cari luas rantau yang dibatasi oleh graf $y = 3^x$, $x + y = 1$ dan $x = 1$.
- (ii) Cari persamaan sebuah bulatan yang melalui fokus dan bucu parabola $y = 8x^2$ dan juga tangen kepada parabola pada titik $(0,0)$.
(30 markah)
- (c) Buktikan bahawa isipadu sebuah kon dengan jejari tapak r dan tinggi h adalah $\frac{1}{3}\pi r^2 h$.
(20 markah)

